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TRANSIENT MIXED CONVECTION ADJACENT TO A VERTICAL FLAT SURFACE

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Abstract—Numerical calculations of the time-varying temperature and velocity profiles are presented for transient aiding mixed convection flow adjacent to a vertical flat surface. The initial condition for the transient is a uniform temperature Blasius flow. An explicit finite-difference scheme was used to calculate time-varying flow and temperature fields which result from two energy input conditions, the sudden generation of a uniform heat flux in the surface material and from a step and uniform change in the temperature of the surface. Transient response was calculated for both surface conditions at Prandtl numbers of 0.72 and 7.6, nominally air and water. For the uniform-flux input condition, response was calculated for values of the surface thermal capacity parameter, Q^*, which correspond to both the one dimensional conduction and the true convection transient regimes. For the uniform, surface temperature condition, the transient always corresponds to the 1-dim. conduction regime. In such a regime, the calculated profiles indicate that both transient temperatures and velocities locally exceed the eventual steady state values. The steady-state profiles for the uniform heat input condition, in air, are in good agreement with the results of previous studies of steady state mixed convection flow.

NOMENCLATURE

- c'', thermal capacity of element per unit surface area;
- g, acceleration of gravity;
- Gr_x, Grashof number, $(g\beta\Delta T x^3/\nu^2)$, dimensionless;
- Gr_x^*, modified Grashof number, $(g\beta q'' x^4/k\nu^2)$, dimensionless;
- k, thermal conductivity;
- L, height of plate;
- Pr, Prandtl number;
- q'', instantaneous energy generation rate per unit of element surface area;
- Q^*, thermal capacity parameter related to the element storage capacity, $c''(g\beta q''\nu^2/k^5)^{1/4}$;
- t, static temperature;
- t_0, instantaneous local plate temperature;
- t_∞, temperature of the undisturbed fluid;
- u, component of velocity in vertical direction;
- v, component of velocity in horizontal direction;
- x, vertical distance above bottom of plate;
- y, horizontal distance from plate.

Greek symbols

- ν, kinematic viscosity;
- ρ, density of fluid;
- $\bar{\tau}$, time;
- τ, non-dimensional time, = TAU.

Subscripts

- 0, at solid–fluid surface;
- ∞, free stream conditions.

1. INTRODUCTION

TRANSIENT mixed convection flows occur in many technological and industrial applications, such as in electronic devices cooled by external forced circulation, or in the core of a nuclear reactor. Mixed convection flow has been studied in the past. However, most of the studies were concerned with steady-state transport. Such studies include those by Sparrow and Gregg [1], Lloyd and Sparrow [2], Oosthuizan and Hart [3], Gryzagoridus [4] and Hommel [5].

Merkin [6] reports the results of an investigation of aiding and opposed mixed convection flows. An isothermal surface immersed in a uniform free stream is studied. For the aiding case, the flow near the leading edge is found to be mostly forced convection flow. However, at large downstream distances, as thermal transport is established, the flow is observed to take on neutral convection characteristics. The solution is obtained by using expansions near the leading edge and far downstream, then closing the gap between these two expansions with a numerical marching scheme. For the opposed flows, however, as buoyancy effects increase, separation is found to occur at large downstream locations, and the analysis is carried out to the point of separation.

A similar study is carried out by Wilks [7], for a surface which is heated by loading it internally with a uniform and constant flux. Expansions are found near the leading edge and far downstream. The solution between these two locations is again determined using a numerical marching scheme.

In a more recent study Carey and Gebhart [8] present the results of a perturbation analysis of the mixed convection flow, adjacent to a vertical uniform flux surface. A matched asymptotic expansion technique is used to construct inner and outer expansions including higher-order boundary layer effects. Experimental measurements of aiding flows in air are found to be in good agreement with the calculations.

This study concerns aiding transient mixed convection flows. A vertical flat surface is assumed to be immersed in an upward flowing uniform stream of ambient fluid, in the boundary layer regime. The steady state Blasius flow profile is then used as an initial uniform temperature flow condition, for the subsequent transient processes which then arise from heating the surface. Two types of heating processes are studied. The first is suddenly loading the surface internally with a uniform and constant flux, q'', as would arise from passing an electric current through conducting surface material of small thickness. For this type of response the thermal capacity of the surface interacts and plays a major role in determining the nature and duration of the ensuing transient flow [9]. The second type of heating process considered here is that which results in a sudden and uniform jump in the surface temperature, to t_0, neglecting the surface thermal capacity. Results are presented for two values of the Prandtl number, namely 0.72 and 7.6, representative of flows in air and in water, respectively.

2. EQUATIONS OF MOTION AND NUMERICAL PROCEDURE

When a flat, vertical, semi-infinite surface is heated while immersed in a uniform stream of fluid, the resulting mixed convection boundary layer flow is represented by the following generalized equations. Equations (1) and (2) embody the Boussinesq approximations where T, defined below, is the buoyancy force.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = T + \frac{\partial^2 U}{\partial Y^2} \tag{2}$$

$$\frac{\partial T}{\partial \tau} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2} \tag{3}$$

These equations are subject to different boundary conditions, depending on the type of heating process under consideration. This necessitates the choice of different non-dimensional groups for each of the heating processes studied. For a uniform input boundary condition at the surface, non-dimensionalization and relevant boundary conditions, for $\tau > 0$, are as follows:

$$T = \frac{t - t_\infty}{(\nu^2 q''^3/g\beta k^3)^{1/4}} = \frac{t - t_\infty}{q''x/k}(Gr_x^*)^{1/4},$$

$$\tau = \frac{\tilde{\tau}}{(k/g\beta q'')^{1/2}} = \frac{\alpha\tilde{\tau}}{x^2}Pr(Gr_x^*)^{1/2}. \tag{4a}$$

$$U = \frac{u}{(\nu^2 g\beta q''/k)^{1/4}} = \frac{ux/\nu}{(Gr_x^*)^{1/4}},$$

$$U_\infty = \frac{u_\infty x/\nu}{(Gr_x^*)^{1/4}} = \frac{u_\infty}{(\nu^2 g\beta q''/k)^{1/4}}. \tag{4b}$$

$$V = \frac{v}{(\nu^2 g\beta q''/k)^{1/4}} = \frac{vx/\nu}{(Gr_x^*)^{1/4}}. \tag{4c}$$

$$X = \frac{x}{(\nu^2 k/g\beta q'')^{1/4}} = (Gr_x^*)^{1/4},$$

$$Y = \frac{y}{(\nu^2 k/g\beta q'')^{1/4}} = \frac{y}{x}(Gr_x^*)^{1/4}, \tag{4d}$$

$$Y = 0: U = V = 0 \tag{4e}$$

$$Y = 0: 1 = Q^*\left(\frac{\partial T}{\partial \tau}\right) - \left(\frac{\partial T}{\partial Y}\right) \tag{4f}$$

$$Y \to \infty: T \to 0,\ U \to \frac{u_\infty x/\nu}{(Gr_x^*)^{1/4}} = \frac{Re_x}{(Gr_x^*)^{1/4}} = U_\infty, \tag{4g}$$

$$X = 0: U = U_\infty. \tag{4h}$$

Condition (4f) represents an energy balance at the solid–fluid interface. The first term on the right hand side is storage in the element in terms of Q^*, while the second term represents convection away by the fluid. Q^* is given by

$$Q^* = c''\left[\frac{\nu^2 g\beta q''}{k^5}\right]^{1/4} = \frac{c''}{\rho c_p x}Pr(Gr_x^*)^{1/4}. \tag{5}$$

On the other hand, for a sudden uniform increase of the surface temperature, from t_∞ to t_0, the non-dimensionalizations and boundary conditions, for $\tau > 0$, are as follows:

$$T = \frac{t - t_\infty}{\Delta T}, \quad \tau = \frac{\tilde{\tau}}{(\nu^{1/2}/g\beta\Delta T)^{2/3}}, \tag{6a}$$

$$U = \frac{u}{(\nu g\beta\Delta T)^{1/3}} = \frac{ux/\nu}{(Gr_x)^{1/3}},$$

$$U_\infty = \frac{u_\infty x/\nu}{(Gr_x)^{1/3}} = \frac{u_\infty}{(\nu g\beta\Delta T)^{1/3}}, \tag{6b}$$

$$V = \frac{v}{(\nu g\beta\Delta T)^{1/3}} = \frac{vx/\nu}{(Gr_x)^{1/3}}, \quad (6c)$$

$$X = \frac{x}{(\nu^2/g\beta\Delta T)^{1/3}} = (Gr_x)^{1/3},$$

$$Y = \frac{y}{(\nu^2/g\beta\Delta T)^{1/3}} = \frac{y}{x}(Gr_x)^{1/3}, \quad (6d)$$

$$Y = 0 : U = V = 0, \quad (6e)$$

$$Y = 0 : T = 1, \quad (6f)$$

$$Y \to \infty : T \to 0,\ U \to \frac{u_{\infty}x/\nu}{(Gr_x)^{1/3}} = \frac{Re_x}{(Gr_x)^{1/3}} = U_{\infty}, \quad (6g)$$

$$X = 0 : U = U_{\infty}. \quad (6h)$$

No similarity formulation is known for these mixed convection transients. The partial differential equations must be solved in their time dependent form. An explicit finite difference scheme is used, wherein the space adjacent to the surface is divided into an $n \times m$ grid in the x and y directions, respectively. Finite difference equations (1) to (3) are solved at each grid point in the flow field, and this solution is marched forward in time.

The initial condition for each kind of transient process was the Blasius flow velocities. These were determined in a transient numerical calculation from the instant of inserting a plate in a uniform flow, by solving equations (1) and (2), subject to boundary conditions (4) or (6), with $T = 0$ in equation (2). The transient numerical calculation then stepped off using the same grid size and node locations. The solution was marched in time till steady flow was reached.

Figure 1 shows the developing uniform temperature Blausius profile, for the u component of velocity. The eventual steady state flow distribution, at TAU = 1000, is also compared to the Blasius similarity solution in Fig. 1. Agreement is found to be quite good, and could be improved by using a finer grid size, n and m. Even though steady state was reached at about $\tau = 500$, the solution is continued to $\tau = 1000$. The resulting final steady state velocity field is then used as an initial condition for solving equations (1)–(3), subject to the boundary conditions (4) or (6).

3. A UNIFORM INPUT SURFACE ELEMENT

When a surface element of appreciable thermal capacity, immersed in an extensive quiescent ambient medium, is suddenly loaded with a step in energy input, several distinctively different flow regimes are found to occur, depending upon the relative thermal capacity of the element and the fluid, c'' and c_p in equation (5). A detailed discussion of these regimes is given in Sammakia and Gebhart [9, 10]. The relevant parameter in determining the regime is the non-dimensional thermal capacity parameter, Q^* in equation (5). Energy dissipated inside the element flows out to the surface by conduction. At short times, a largely 1-dim. conduction field extends out into the adjacent fluid. This temperature increase causes buoyancy, which in turn generates only one component of velocity, parallel to the surface. This 1-dim. process lasts at any downstream location until the entrainment effects beginning near the leading edge, reach that location. Eventually steady-state is reached. The extent and duration of this 1-dim. regime depends upon the value of the non-dimensional thermal capacity parameter Q^*. Under some circumstances, local temperature and velocity levels exceed the final steady state

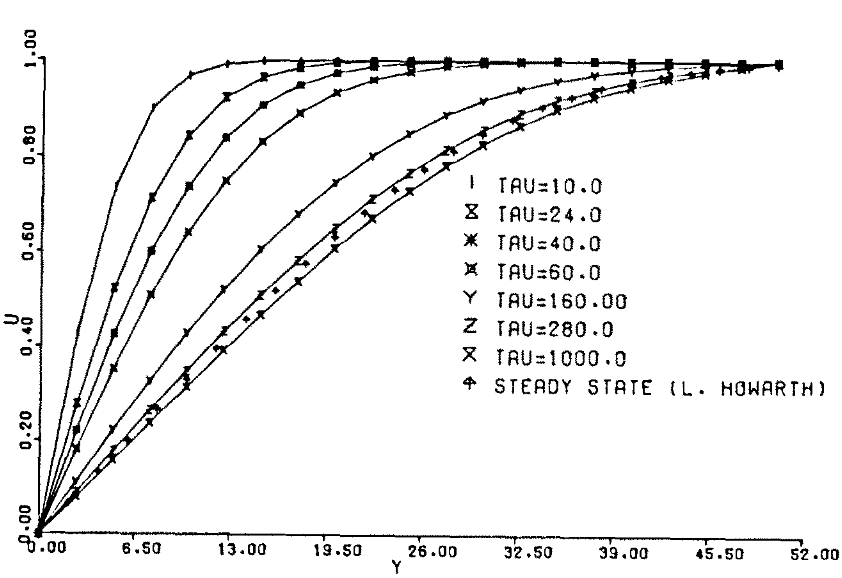

FIG. 1. Transient velocity profiles for forced convection flow adjacent to a flat semi-infinite surface.

value during the transient process. This 1-dim. response mechanism essentially dominates the whole transient when the surface thermal capacity is small, say for $Q^* < 1$ in quiescent air and $Q^* < 5.9$ in water [9, 10].

All the present computations for a uniform input condition are for $X = 0$ to 100, where $X = 100$ in equation (4d) corresponds to $Gr_x^* = 10^8$, a laminar boundary layer flow, as described in Qureshi and Gebhart [11]. The additional parameter, in mixed convection, is the free stream velocity U_f. In the present study U_f is taken as 1. It will be shown that this results in flows which are eventually more influenced by natural than forced convection effects. That is, as steady state is reached, the maximum velocity in the boundary region is considerably higher than the free stream velocity.

Figures 2 and 3 show the developing temperature and velocity profiles at $X = 100$ for $Pr = 0.72$, and $Q^* = 0.1$. This value of Q^* corresponds to a 1-dim. conduction regime of flow in air. Figure 2 shows four transient temperature profiles and the eventual steady state temperature distribution. At $\tau = 14$ the temperature locally exceeds the steady state value everywhere in the flow field. In Fig. 3 a corresponding velocity overshoot is observed at $\tau = 14$, although the velocity exceeds the steady state value only near the maximum velocity region.

The distance out from the surface at which the velocity reaches the free stream velocity, $U_f = 1$, represents the edge of the viscous boundary layer. The distances found here are comparable to the calculated thermal boundary layer thickness for $Pr = 0.72$, for pure natural convection. This similarity to pure natural convection flow arises because U_f is small compared to the velocity levels, which arise purely due to natural convection effects.

As indicated in the first section, Carey and Gebhart [8] obtained new and more complete expansion solutions for steady-state mixed convection adjacent to a vertical uniform-heat flux surface. The results are valid at moderate downstream distances where the flow resembles a purely free convection flow pertubed by a non-zero free stream velocity and non-boundary layer effects. The expansions for the u velocity and local temperature are

$$u = U_{CG}[F'_0(\eta) + \varepsilon_M F'_1(\eta) + \varepsilon_H F'_2(\eta) + \ldots]$$

and

$$t - t_f = \Delta T[H_0(\eta) + \varepsilon_M H_1(\eta) + \varepsilon_H H_2(\eta) + \ldots]$$

where

$$U_{CG} = \frac{5\nu}{x}\left(\frac{Gr^*}{5}\right)^{1.5}, \Delta T = \frac{q''x}{k}\left(\frac{Gr^*}{5}\right)^{-1.5},$$

$$\eta = \frac{y}{x}\left(\frac{Gr^*}{5}\right)^{1.5}$$

and

$$\varepsilon_M = Re_x(Gr^*/5)^{-2.5}, \varepsilon_H = (Gr^*/5)^{-1.5}.$$

Here ε_M and ε_H are the perturbation parameters associated with mixed convection and non-boundary layer effects, respectively. The velocity and temperature functions, $F'_i(\eta)$ and $H_i(\eta)$, were found for $Pr = 0.733$ and 6.7. These results convert to the present formulation of U, T and Y as follows:

$$U(Y) = 5^{3.5}(Gr^*)^{3.20}[F'_0(\eta) + \varepsilon_M F'_1(\eta) + \varepsilon_H F'_2(\eta) + \ldots],$$

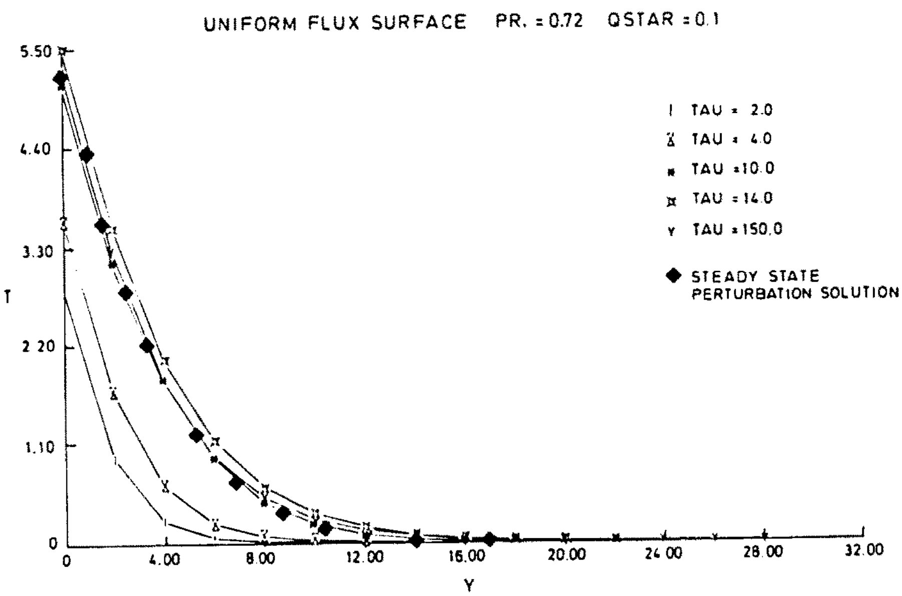

FIG. 2. Transient temperature profiles adjacent to a uniform flux surface, immersed in a uniform stream of air.

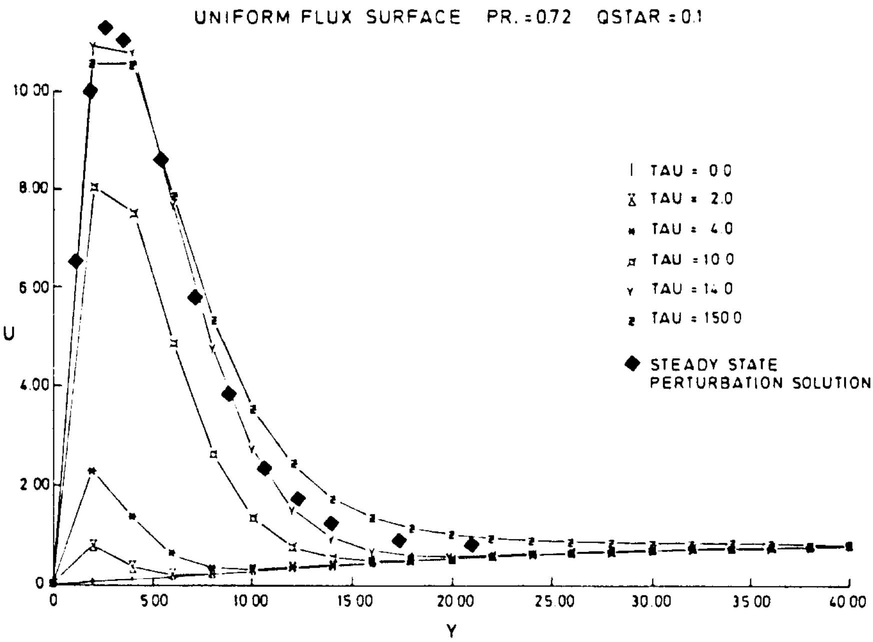

FIG. 3. Transient velocity profiles adjacent to a uniform flux surface immersed in a uniform stream of air.

$$T(Y) = 5^{1/5}(Gr^*)^{1/20}[H_0(\eta) + \varepsilon_M H_1(\eta) + \varepsilon_H H_2(\eta) + \dots],$$

$$Y = 5^{1/5}(Gr^*)^{1/20}\eta.$$

Points from the results of Carey and Gebhart [8], for $Pr = 0.733$ are thus plotted in Fig. 2 and 3. The steady-state results of the present analysis are in fairly good agreement with the perturbation analysis results, despite the spatial grid used in the present computations.

Transient response for $Pr = 7.6$, at $Q^* = 2.0$, is shown in Figs. 4 and 5. For this value of Q^* the transient response also lies in the 1-dim. conduction regime, in water. Here, also, as for air, both earlier temperature and velocity levels locally slightly exceed the final steady state distributions. In Fig. 4 the overshoot is seen to occur at $\tau = 42$. Figure 5 shows that the uniform stream velocity U_∞ is approximately 25% of the maximum velocity level achieved in steady state. The viscous boundary layer in Fig. 4 is seen to quickly become much thicker than the thermal layer, as expected for $Pr = 7.6$. During the transient, the velocity early exceeds U_∞ near the surface, where the

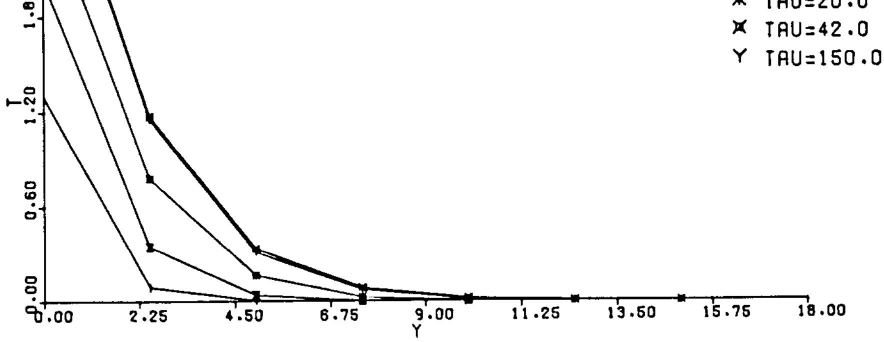

FIG. 4. Transient temperature profiles adjacent to a uniform flux surface immersed in a uniform stream of water.

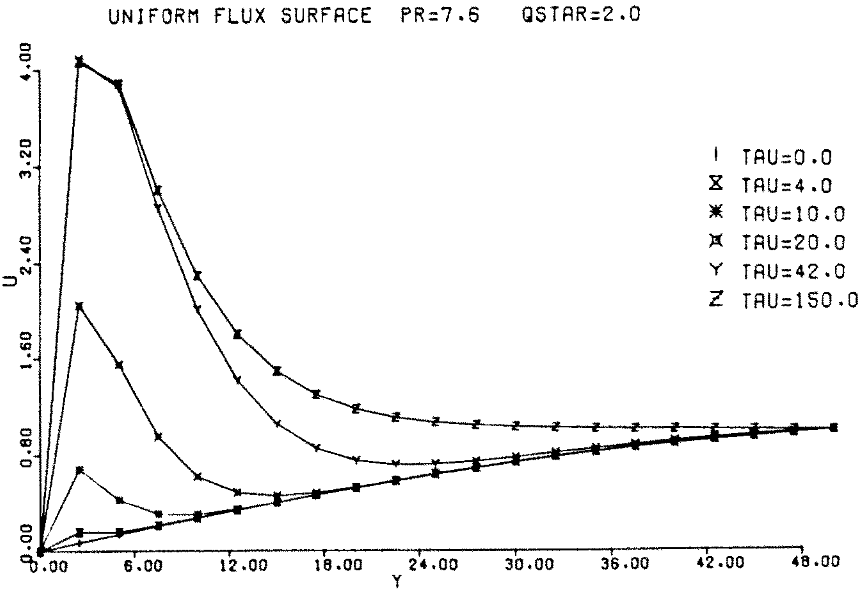

FIG. 5. Transient velocity profiles adjacent to a uniform flux surface immersed in a uniform stream of water.

buoyancy force is strongest. Thus the transient profiles resemble a natural convection flow near the surface, matched to a Blasius velocity profile away from the surface. However, the final steady state profile drops uniformly from the maximum velocity near the surface to the free stream velocity at the outer edge of the viscous boundary layer.

The true convection transients, where all the terms in the equations are of comparable order of magnitude, occur for surfaces with intermediate thermal capacities. The range in pure natural convection flows are $1 \leq Q^* \leq 10$ for air and $5.9 \leq Q \leq 59$ for water. Values beyond 10 for air and 59 for water correspond to quasi-static response. That is, eventual steady state is reached uniformly through a succession of quasi-static processes characteristic of the instantaneous value of t_0. Under such circumstances, local values of temperature and velocity during the transient do not exceed their steady-state values.

Figures 6 and 7 show response in air for $Q^* = 5.0$, which would be a true convection transient in pure natural convection. A similar transient regime response in water, for $Q^* = 30.0$, is shown in Figs. 8 and 9. In both air and water, the temperature and velocity

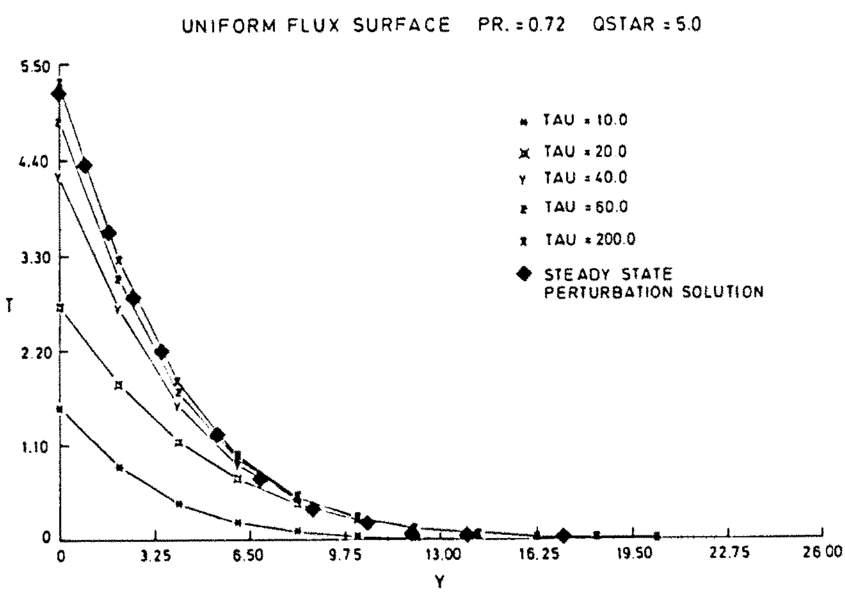

FIG. 6. Transient temperature profiles adjacent to a uniform flux surface immersed in a uniform stream of air.

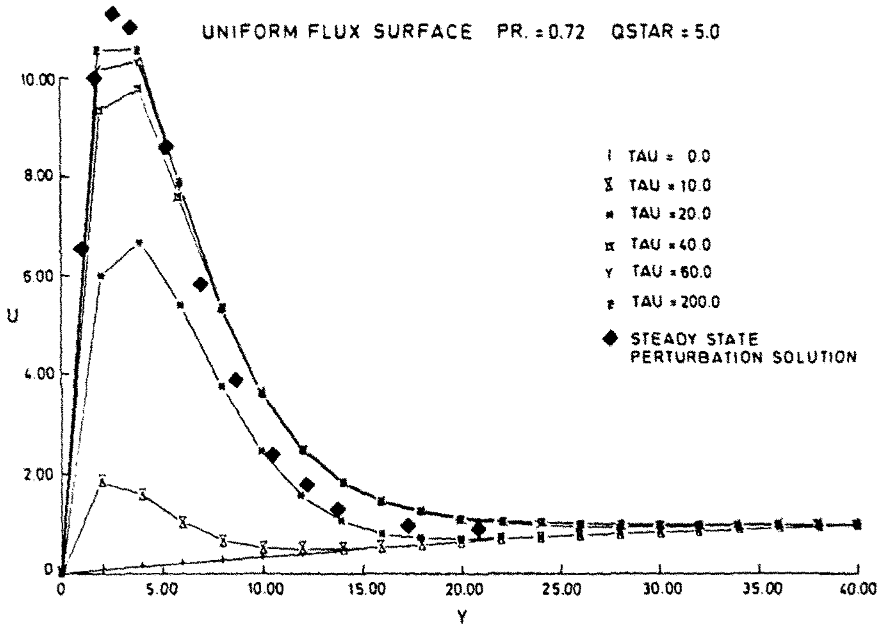

FIG. 7. Transient velocity profiles adjacent to a uniform flux surface immersed in a uniform stream of air.

levels do not exceed the eventual steady state levels during the transient response. The temperature and velocity levels are higher in air than in water, for the same $Gr_x^* = 10^8$. Also, the thermal boundary layer thickness in water is much smaller than that of air. For air, the steady state results are in good agreement with the values calculated by Carey and Gebhart [8], which are also plotted in Figs. 6 and 7.

4. ISOTHERMAL SURFACE CONDITION

For this circumstance, the temperature of the surface is assumed to be suddenly and uniformly raised at the start of the transient process, and then held constant at this level. This in effect neglects the effect of the surface thermal capacity on the transient process. The only remaining parameters in the model are the Prandtl number, the Grashof number, and the free stream velocity. For such a boundary condition in purely natural convection flows, Hellums and Churchill [12] have shown that, for $Pr = 0.72$, a temporal minimum in the heat transfer coefficient arises. The accuracy of this result was supported by comparing it

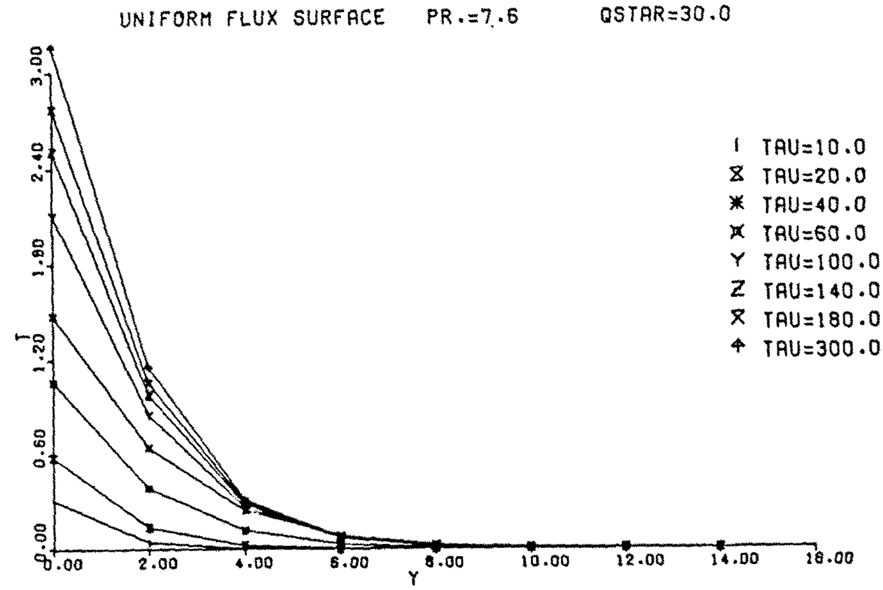

FIG. 8. Transient temperature profiles adjacent to a uniform flux surface immersed in a uniform stream of water.

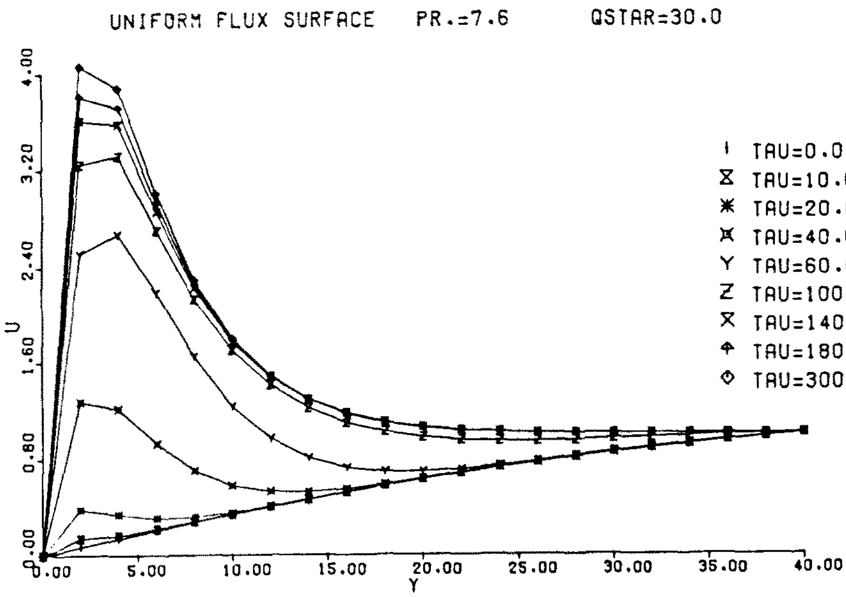

FIG. 9. Transient velocity profiles adjacent to a uniform flux surface immersed in a uniform stream of water.

to the analytical conduction solution, during the early transient. Such behavior may be expected to result from the 1-dim. nature of the convection process during the early transient. Our eventual steady state values were also found to be in good agreement with solutions obtained using the similarity formulation.

In this study, the same two values, $Pr = 0.72$ and 7.6, are investigated. The free stream velocity is again taken as $U_f = 1.0$. Figures 10 and 11 show the response for $Pr = 0.72$. At $\tau = 25$ the temperature level was to locally exceed the eventual steady state values everywhere in the flow field, except at the surface of course. However, the excess is too small to be seen on Fig. 10. The corresponding velocity profile at $\tau = 25$ also exceeds the steady state value in the region near where the maximum velocity occurs, visible in Fig. 11.

Figures 12 and 13 show the transient response for $Pr = 7.6$. The temperature at $\tau = 45$ considerably exceeds the final steady state values, more than for $Pr = 0.72$. The relatively thermal thinner thermal boundary layer, at $Pr = 7.6$ mostly lies deep within the viscous layer. The Nusselt number at $\tau = 45$ is found to be about 5%

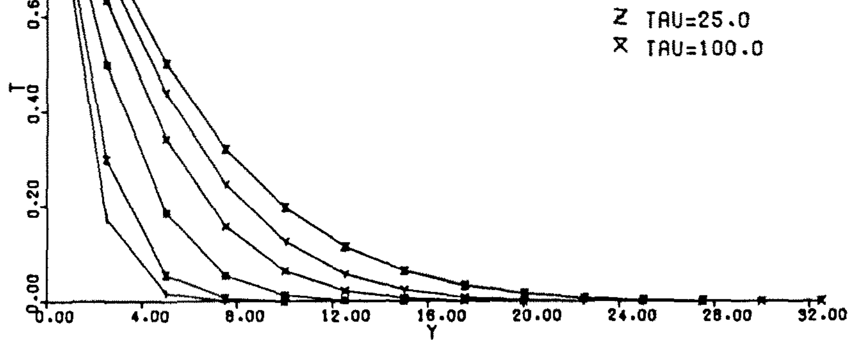

FIG. 10. Transient temperature profiles adjacent to an isothermal surface immersed in a uniform stream of air.

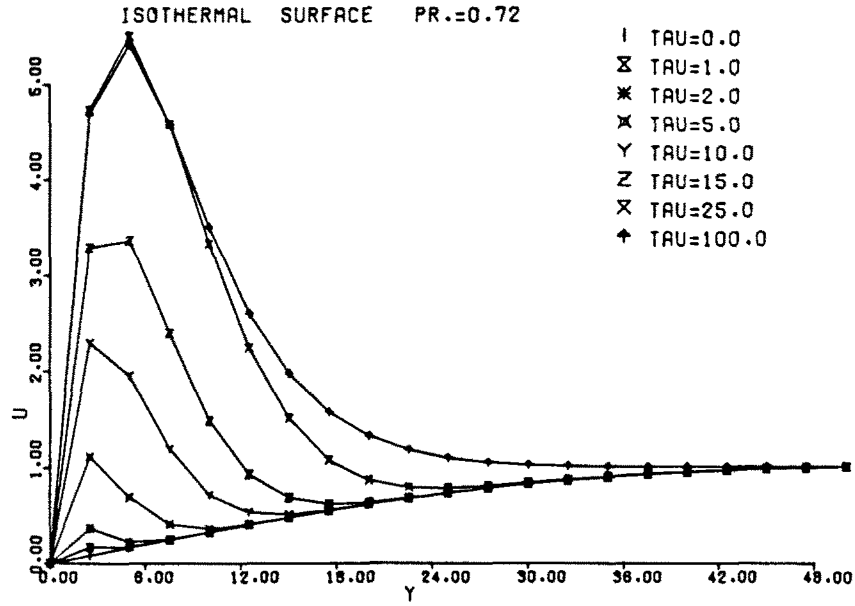

FIG. 11. Transient velocity profiles adjacent to an isothermal surface immersed in a uniform stream of air.

lower than that at steady state. The velocity response, again at $\tau = 45$, shows a very slight overshoot in a very narrow region near the surface.

5. CONCLUSIONS

The present investigation deals with transient response in aiding mixed convection flow adjacent to a flat vertical surface. The transient forced convection flow, wherein a flat surface is suddenly immersed in a uniform stream of fluid is first solved using a finite-difference scheme, marching until steady state is reached. These calculations are in good agreement with those from the similarity formulation. This steady state velocity field is then used as an initial condition when the buoyancy effects arise in the transient mixed convection flow.

Two types of boundary conditions were considered, a uniform input to a surface element and a uniform surface temperature jump. Two values of the Prandtl number are studied for each of these conditions, $Pr = 0.72$ and $Pr = 7.6$, as for air and water. For the uniform energy input, the thermal capacity of the surface plays

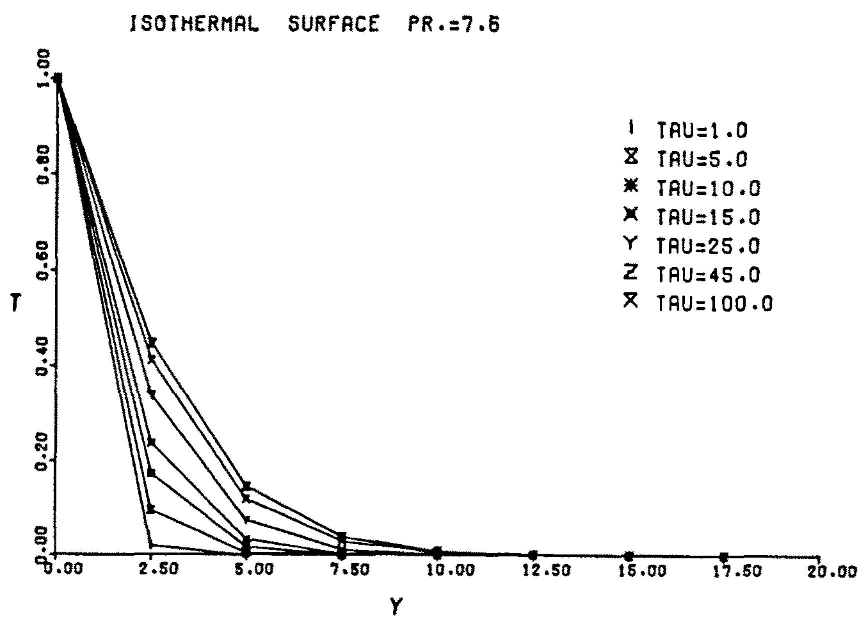

FIG. 12. Transient temperature profiles adjacent to an isothermal surface immersed in a uniform stream of water.

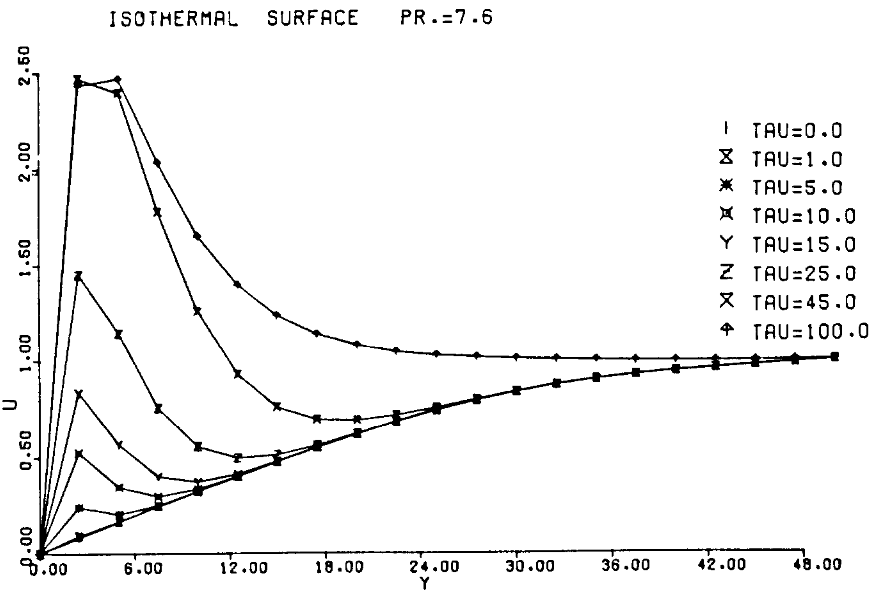

FIG. 13. Transient velocity profiles adjacent to an isothermal surface immersed in a uniform stream of water.

a major role in determining both the resulting transient regimes. The element thermal capacity is also found to affect response as well as to determine the duration of the transient process.

In this study two values of the non-dimensional thermal capacity parameter, Q^*, are investigated for each Prandtl number. These values of Q^* are chosen to correspond to a 1-dim. regime and to a true convection transient regime. In the 1-dim. regime, both transient temperatures and velocities locally exceed the eventual steady states. For the uniform-flux surface, the calculated steady-state profiles are in good agreement with those predicted by the perturbation analysis of Carey and Gebhart [8].

The same values for the Prandtl number are investigated for a jump to an isothermal surface condition. Then the surface thermal capacity plays no role. The only resulting type of transient is the 1-dim. conduction regime wherein both the transient temperatures and velocities overshoot their steady state values at some locations in the boundary layer. For $Pr = 7.6$ the thermal boundary layer thickness is found to be much smaller compared to the viscous boundary layer. This is in contrast to the case of $Pr = 0.72$ where both layers are of comparable thickness.

Finally, it is of interest to note that, for a forced convection flow, the velocity component normal to the surface, v, acts away from the surface. This moves ambient fluid away from the boundary region. On the other hand, in pure natural convection flows, this entrainment component v acts in the opposite direction, thereby entraining fluid into the boundary layer. Thus, at the beginning of the mixed convection transient, the v component of velocity has its highest positive value. As the surface warms and the natural convection induced flow increases, the value of v decreases. If the flow is more strongly natural convection than forced, as in these calculations, the v component of velocity is found to further decrease to negative everywhere, as steady-state is eventually reached.

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CONVECTION MIXTE TRANSITOIRE ADJACENTE A UNE SURFACE PLANE ET VERTICALE

Résumé—Des calculs numériques des profils variables de température et de vitesse sont présentés pour une convection mixte sur une surface plane, verticale. La condition initiale est un écoulement de Blasius à température uniforme. On utilise un schéma explicite à différences finies pour calculer les champs variables de température et de vitesse qui résultent des deux conditions d'entrée de l'énergie : génération soudaine d'un flux thermique uniforme à la surface du matériau et un changement uniforme en échelon de température pariétale. La réponse transitoire est calculée pour les deux conditions à des nombres de Prandtl de 0,72 et 7,6 correspondant à l'air et l'eau. Pour la condition de flux uniforme, la réponse est calculée pour des valeurs du paramètre de capacité surfacique Q^* qui correspondent à la fois à la conduction monodimensionnelle et aux vrais régimes de convection transitoires. Pour la condition de température pariétale, le transitoire correspond toujours au régime de convection monodimensionnel. Dans un tel régime, les profils calculés montrent que les températures et les vitesses dépassent localement les valeurs éventuelles de l'état permanent. Les profiles d'état permanent pour la condition de flux dans l'air sont en bon accord avec les résultats des études antérieures de la convection mixte permanente.

INSTATIONÄRE GEMISCHTE KONVEKTION AN EINER SENKRECHTEN WAND

Zusammenfassung—Es werden numerische Berechnungen der zeitlich veränderlichen Temperatur- und Geschwindigkeitsprofile von instationären gemischten Konvektionsströmungen an einer senkrechten Wand vorgelegt. Die Anfangsbedingung für den instationären Verlauf ist eine Strömung nach Blasius mit einheitlicher Temperatur. Ein explizites Differenzenverfahren wurde angewandt, um die zeitlich veränderlichen Strömungs- und Temperaturfelder zu berechnen, die aus zwei Randbedingungen für die Energiezufuhr resultieren: aus der plötzlichen Erzeugung eines einheitlichen Wärmestromes von der Wand und aus einer schrittweisen und einheitlichen Änderung der Wandtemperatur. Die Übergangsfunktionen wurden für beide Randbedingungen für Prandtl-Zahlen von 0,72 und 7,6 bzw. Luft und Wasser berechnet. Für die Randbedingung des einheitlichen Wärmestromes wurde die Übergangsfunktion mit dem Wärmekapazitätsparameter der Wand, Q^*, berechnet, die sowohl für die Bedingungen der instationären eindimensionalen Wärmeleitung als auch der echten Konvektion gelten. Für die Randbedingung der konstanten Oberflächentemperatur entspricht der instationäre Verlauf immer dem Vorgang der eindimensionalen Wärmeleitung. Unter solchen Betriebsbedingungen zeigen die berechneten Profile, daß sowohl Temperaturen wie Geschwindigkeiten örtlich die Grenzwerte des stationären Zustandes übersteigen. Die stationären Profile unter der Bedingung des konstanten Wärmestromes stimmen bei Luft gut mit den Ergebnissen früherer Untersuchungen der stationären Strömung bei gemischter Konvektion überein.

НЕСТАЦИОНАРНАЯ СМЕШАННАЯ КОНВЕКЦИЯ У ВЕРТИКАЛЬНОЙ ПЛОСКОЙ ПОВЕРХНОСТИ

Аннотация — Представлены численные расчеты профилей нестационарных температуры и скорости при нестационарной смешанной конвекции у вертикальной плоской поверхности. Начальным условием переходного процесса служит течение Блазиуса с постоянной температурой. Для расчета нестационарных динамических и температурных полей, обусловленных двумя видами подвода энергии, скачкообразным выделением однородного теплового потока на поверхности, а также ступенчатым и равномерным изменениями температуры поверхности, использовалась явная конечно-разностная схема. Переходный процесс рассчитывался при значениях числа Прандтля, равных 0,72 и 7,6, характерных для воздуха и воды. При однородном тепловом потоке переходный процесс рассчитывался для значений теплоемкости поверхности Q^*, соответствующих как одномерной теплопроводности, так и чисто конвективному нестационарному режиму. При однородной температуре поверхности переходный процесс всегда соответствует одномерному режиму теплопроводности. При таком режиме рассчитанные профили свидетельствуют о том, что локальные значения как нестационарной температуры, так и скорости существенно выше соответствующих стационарных значений. Стационарные профили в воздухе при однородном тепловом потоке хорошо согласуются с результатами ранее проведенных исследований стационарной смешанной конвекции.